# General theory of detrending-operation-based scaling analysis methods and applications

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Detrending-operation-based analysis methods such as detrending fluctuation analysis (DFA) and detrending moving average analysis (DMA) have become widely used to characterize long-range correlations and fractal scaling behaviour in nonstationary time series. Here we present a general framework by the example of DFA to facilitate the understanding of the mathematical foundations of such scaling analysis methods. DFA is based on the random walk theory and provides the fluctuation function which is the square root of the mean square deviations of the integrated time series substracted by a polynomial trend averaged over time windows of the same length. In our framework DFA can be described as mean square displacement statistics estimated by detrending-operation-based analysis with a displacement as weighted partial sum of the original series. Using this background we can derive a rigorous relationship between the fluctuation function of DFA and the autocorrelation function. The fluctuation function is an integral transform of the autocorrelation function with detrending kernel working as filter. This fluctuation function can be estimated via segmentation of the time axis just as it is originally constructed in DFA. With the relationship between the fluctuation function and autocorrelation function we can derive analytical solutions of the fluctuation function for short-range and long-range correlated processes. Thereby we investigate the long known problem of the crossover behaviour in the scaling of short-range correlated processes and provide an analytical expression of the crossover point which depends on the detrending order of the method and the characteristic correlation time of the process. This is important for practical applications where usually complicated trends exist. Furthermore our framework provides a tool to understand the detrending operation in the case of intrinsic nonstationarities such as fractional Brownian motion (FBM). By the construction of DFA only external nonstationarities are removed naturally. Nevertheless DFA with linear detrending also gives the correct scaling exponent for FBM. Our framework provides a unified picture of the detrending operation for both extrinsic nonstationarities such as trends and intrinsic nonstationarites such as fractional Brownian motion (FBM).

[l] M. Höll, H. Kantz, Eur. Phys. J. B **88**, 126 (2015).

[2] M. Höll, H. Kantz, Eur. Phys. J. B **88**, 327 (2015).

[3] M. Höll, Y. Zhou, H. Kantz, Phys. Rev. E **94**, 042201 (2016).